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A SINGLE ACCEPTANCE SAMPLING PLAN FOR TRUNCATED LIFE TEST HAVING THE (P-A-L) EXTENDED WEIBULL DISTRIBUTION

M. S. Hamed

Department of Administrative Sciences
Faculty of Business Administration in Khaybar
Taibah University
Kingdom of Saudi Arabia

Department of Applied Statistics
Faculty of Commerce
Benha University
Egypt
e-mail: moswilem@gmail.com

Abstract

The paper proposes the single acceptance sampling plan under the time truncated life test when the product lifetime follows a (P-A-L) extended Weibull distribution. The motive of this paper is to design the parameters (such as the sample size and acceptance number) satisfying both the producer's risk and consumer's risks simultaneously, at the specified quality levels, while the termination time and the number of testers are specified. Eventually, the result are explained with tables and examples.

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1. Introduction

Accepting sampling plans is concerned with inspection and decision making regarding product, one of the oldest aspects of quality assurance. The acceptance sampling plans are concerned with acceptance or rejection of a submitted lot of size of products on the basis of the quality of products inspected in a sample taken from the lot. An acceptance sampling plan is specified plan that establishes the minimum sample size to be used for testing. In most of acceptance sampling plans for a truncated life test, the major issue is to determine the sample size from the lot under consideration. The ordinary acceptance sampling plans for different distributions have been developed by many researchers including Kantam et al. [12], Baklizi [1], Balakrishnan et al. [11] and Lio et al. [13, 14]. However, it requires more cost, time and supervision to gather the sample for making a decision of either accepting or rejecting the lot of products. Therefore, the ordinary acceptance sampling plan is usually expensive to be implemented. On the other hands, in group sampling case, the experimenter can reduce the experiment expenditure by installing more than on item on a single tester. The other advantage of the group plan is that it provides more strict inspection of the product than the ordinary acceptance sampling because sample is distributed over than one group. The condition of acceptance is applied to each group for a better product inspection. Therefore, the group acceptance sampling plan (GASP) has attracted researchers due to its advantage over the ordinary acceptance sampling plan under a truncated life test. The sudden death life testing is always performed in GASP. Jun et al. [3] proposed variables sampling plans for the Weibull distribution under the sudden death testing. Various GASPs under a time truncated life test are available in literature. For example, Aslam and Jun [8] studied GASP for the inverse Rayleigh and the log-logistic distributions. Aslam and Jun [7] proposed the GASPs for the Weibull distribution and Aslam et al. [9] proposed the plans for the gamma distribution. More recently, Aslam et al. [10] proposed a modified group plan for the Weibull distribution.

The purpose of this paper is to propose a SASP based on truncated life tests when the lifetime of a product follows the (P-A-L) extended Weibull distribution. Further, we obtain the sample size and the acceptance number simultaneously for given values of both risks using the two point approach. The rest of the paper is organized as follows: we will introduce the glance about the (P-A-L) extended Weibull distribution in Section 2. The proposed SASP along with the operating characteristics is described in Section 3. The results are explained with some examples in Section 4. Then we conclude with some remarks in Section 5.

2. The (P-A-L) Extended Weibull Distribution

The (P-A-L) extended Weibull distribution can be used for the life modelling in reliability analysis, life testing problems and acceptance sampling plan. The (P-A-L) extended Weibull distribution was derived by Ahmed and Fattah [2] to generalize the Weibull probability distribution. The probability density function (pdf) and the cumulative distribution function (cdf) of the (P-A-L) extended Weibull distribution, respectively, are given by:

$$f(t, \delta, \sigma, \rho, \nu) = \frac{1}{\ln \rho}\left[\frac{(\rho - 1)\frac{\nu\delta}{\sigma}\left(\frac{t}{\sigma}\right)^{\delta-1} e^{-\left(\frac{t}{\sigma}\right)^{\delta}}}{\left[1-(1-\rho\nu)e^{-\left(\frac{t}{\sigma}\right)^{\delta}}\right]\left[1-(1-\nu)e^{-\left(\frac{t}{\sigma}\right)^{\delta}}\right]}\right];$$

$$t > 0,\ \delta,\ \sigma,\ \nu > 0,\ \rho > 1, \tag{1}$$

$$F(t, \delta, \sigma, \rho, \nu) = 1 - \frac{1}{\ln \rho}\ln\left[\frac{1-(1-\rho\nu)e^{-\left(\frac{t}{\sigma}\right)^{\delta}}}{1-(1-\nu)e^{-\left(\frac{t}{\sigma}\right)^{\delta}}}\right]. \tag{2}$$

It is clear that the (P-A-L) extended Weibull distribution is very flexible (see Ahmed and Fattah [2]). This is so since several other distributions follow as special cases from the (P-A-L) extended Weibull distribution, by selecting the appropriate values of the parameters. These special cases include eleven distributions.

The mean of the (P-A-L) extended Weibull distribution is given by:

$$E(T) = \frac{\sigma\nu(\rho - 1)}{\ln\rho}\,\Gamma\left(\frac{1}{\delta} + 1\right)\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\frac{(1-\rho\nu)^{i}(1-\nu)^{j}}{(1+i+j)^{\left(\frac{1}{\delta}+1\right)}}. \tag{3}$$

The median of the (P-A-L) extended Weibull distribution is given by:

$$median = \sigma\left\{-\ln\left[\frac{1 - e^{0.5}\rho}{1 + (e^{0.5}\nu - \nu - e^{0.5})\rho}\right]\right\}^{\frac{1}{\delta}}. \tag{4}$$

3. Design of SASP for the (P-A-L) Extended Weibull Distribution

Let μ represent the true value of the median of lifetime distribution of a product and μ_0 denote the specified median, under the assumption that lifetime of an item follows a (P-A-L) extended Weibull distribution. We interested in designing a sampling plan in order to assure that the median life of items in a lot (μ) is greater than the specified life (μ_0). We will accept the lot if there is enough evidence that $\mu \geq \mu_0$ at certain levels of consumer's and producer's risks. Otherwise, we have to reject the lot.

Let us propose the following single acceptance sampling plan based on the truncated life test (Stephens [6]):

Implementation of single attribute sampling plan is very simple. It involves taking a random sample of size n from a lot of size N. The number of defectives (or defects) d found is compared to an acceptance number c after experiment time (the termination time) t_0. If the number found is less than or equal to c, the lot is accepted. If the number found is greater than c, the lot is rejected. We are interested in determining the sample size *n* and the action limit *c* which satisfies both the risks at the same time, whereas the termination time t_0 is assumed to be specified.

Suppose that the life time of a product follows the (P-A-L) extended Weibull distribution which has the cdf given by equation (2). As pointed out by Grant and Leavenworth [4] and Stephens [6], binomial distribution can be used to express the OC curve for a sampling plan when the lot size is large enough and the experimenter focuses only on two options either to accept or reject the lot. The lot of products is accepted only if the number of tolerance *c* is equal or less than the number of failures *d* in the sample *n*, so the lot acceptance probability will be

$$L(P) = \sum_{i=0}^{c} \binom{n}{i} P^i (1-P)^{n-i}, \tag{5}$$

where P is the probability that any item in sample fails before the termination time t_0. It would be convenient to determine the termination time t_0 as a multiple of specified life μ_0. That is, we will consider $t_0 = a\mu_0$ for a constant *a*, for example, $a = 0.5$ means that the experiment time is just half of the specified life and *P* is given by:

$$P = F(t_0) = 1 - \frac{1}{\ln \rho} \ln\left[\frac{1-(1-\rho\nu)e^{-\left(\frac{a.m}{\mu/\mu_0}\right)^{\delta}}}{1-(1-\nu)e^{-\left(\frac{a.m}{\mu/\mu_0}\right)^{\delta}}}\right], \tag{6}$$

where $m = \left\{-\ln\left[\frac{1-e^{0.5}\rho}{1+(e^{0.5}\nu - \nu - e^{0.5})\rho}\right]\right\}^{\frac{1}{\delta}}$ and $m = \mu/\sigma$ from equation (3).

For the known values of the parameters δ, λ and ν, *P* can be evaluated when the multiplier *a* and the ratio μ/μ_0 are specified. We can express the quality level of a product in terms of ratio of its median lifetime to the specified life μ/μ_0. The probability of rejection a good lot is called the *producer's risk*, whereas the probability of accepting a bad lot is known as the *consumer's risk*. The consumer demands that the lot acceptance probability should be smaller than the specified consumer's risk β at a lower quality level (usual at ratio 1), whereas the producer requires that the lot rejection probability should be smaller than the specified producer's α at higher quality level. When the quality level is expressed by the ratio mentioned earlier, the proposed two-point approach of finding the design parameters is to determine the sample size and the acceptance number that satisfy the following two inequalities simultaneously:

$$L(P\backslash\mu/\mu_0 = r_1) \leq \beta, \tag{7}$$

$$L(P\backslash\mu/\mu_0 = r_2) \geq 1 - \alpha, \tag{8}$$

where r_1 is the mean ratio at the consumer's risk and r_2 is the mean ratio at the producer's risk. Let p_1 be the failure probability corresponding to the consumer's risk and p_2 be the failure probability corresponding to be producer's risk, the minimum sample size and action number required can be determined by considering the consumer's risk and producer's risk at the same time through the following inequalities:

$$L(p_1) = \sum_{i=0}^{c} \binom{n}{i} p_1^i (1 - p_1^i)^{n-1} \leq \beta, \tag{9}$$

$$L(p_2) = \sum_{i=0}^{c} \binom{n}{i} p_1^i (1 - p_1{}^i)^{n-1} \geq 1 - \alpha. \tag{10}$$

The design parameter in terms of integers can be found by using a search, which can be implemented by Matlab program.

Table 1 shows the minimum sample size and the acceptance number required for the proposed single sampling plan according to various values of the consumer's risk $(\beta = 0.25, 0.10, 0.05, 0.01)$ when the true median equals the specified life and 5% of producer's risk when the true median is $(r_2 = 2, 4, 6, 8, 10)$ times the specified life. Two levels of the test termination time multiplier $(a = 0.5, 0.7)$ are considered. We considered two values of the shape parameter of (P-A-L) extended Weibull distribution $(\delta = 2, 4)$ with other fixed parameters $(\rho = 0.6$ and $\nu = 2)$.

Table 1. Minimum sample size and acceptance number for SASP when $(\delta = 2, \nu = 2, \rho = 0.6)$

β	$\mu/\mu_0 = r_2$	$a = 0.5$			$a = 0.7$		
		n	c	$L(P_2)$	n	c	$L(P_2)$
0.25	2	9	3	0.95186	5	3	0.96538
	4	4	1	0.9894	3	1	0.98007
	6	2	0	0.96165	1	0	0.96223
	8	2	0	0.97829	1	0	0.97866
	10	2	0	0.98606	1	0	0.98632
0.10	2	15	5	0.97251	9	5	0.97036
	4	6	1	0.97498	3	1	0.98007
	6	6	1	0.99466	3	1	0.99583
	8	3	0	0.96761	2	0	0.95778
	10	3	0	0.97917	2	0	0.97282
0.05	2	16	5	0.96209	11	6	0.97416
	4	7	1	0.96597	4	1	0.96236
	6	7	1	0.99262	4	1	0.99187
	8	4	0	0.95705	2	0	0.95778
	10	4	0	0.97232	2	0	0.97282
0.01	2	24	7	0.96441	14	7	0.96207
	4	12	2	0.9867	7	2	0.98404
	6	9	1	0.98767	5	1	0.98678
	8	9	1	0.99592	5	1	0.99564
	10	6	0	0.95877	5	0	0.95951

Table 2. Minimum sample size and acceptance number for SASP when $(\delta = 4, \nu = 2, \rho = 0.6)$

β	$\mu/\mu_0 = r_2$	$a = 0.5$			$a = 0.7$		
		n	c	$L(P_2)$	n	c	$L(P_2)$
0.25	2	16	1	0.98708	5	1	0.98409
	4	8	0	0.99454	2	0	0.99475
	6	8	0	0.99892	2	0	0.99896
	8	8	0	0.99966	2	0	0.99967
	10	8	0	0.99986	2	0	0.99987
0.10	2	22	1	0.97618	6	1	0.9768
	4	13	0	0.99114	3	0	0.99214
	6	13	0	0.99824	3	0	0.99844
	8	13	0	0.99944	3	0	0.99951
	10	13	0	0.99977	3	0	0.9998
0.05	2	27	1	0.96508	7	1	0.9684
	4	17	0	0.98843	4	0	0.98954
	6	17	0	0.99771	4	0	0.99793
	8	17	0	0.99927	4	0	0.99934
	10	17	0	0.9997	4	0	0.99973
0.01	2	48	1	0.98439	12	2	0.98807
	4	26	0	0.98237	6	0	0.98434
	6	26	0	0.99649	6	0	0.99689
	8	26	0	0.99889	6	0	0.99901
	10	26	0	0.99954	6	0	0.9996

In these tables, note that as the ratio r_2 increases, the samples size and the acceptance numbers decrease at the same time. We need a smaller number of sample size if the termination ratio increases. For example, from Table 1, if a changes from 0.5 to 0.7 at $\beta = 0.05$, then the sample size has been changed from $n = 16$ to $n = 5$ when $n_2 = 2$.

4. Description of Tables and Examples

Suppose, for example that the lifetime of a product follows a (P-A-L) extended Weibull distribution with the shape parameter $\delta = 2$ and other parameters are $\rho = 0.6$ and $\nu = 2$. Suppose that it is desired to design a single sampling plan to assure that the median life is greater than 1000h through the experiment to complete by 1000h using testers equipped with three products each. It is assumed that the consumer's risk is 1% when the true median is 1000h and the producer's risk is 5% when the true median is 2000h. Since $\beta = 0.25$, $a = 0.05$ and $r_2 = 2$, the minimum sample size and acceptance number can be obtained as $n = 16$ and $c = 1$ from Table 2. We will accept the lot if no more than one failure occurs before 1000h in the sample. For this proposed sampling plan under the (P-A-L) extended Weibull distribution, the sample size decreases and the OC values increase as follows when the true median increases. This is summarized from Table 2.

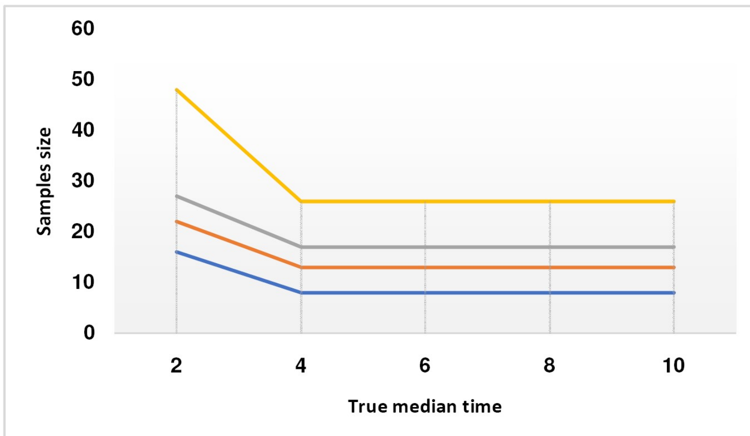

Figure 1. True median time versus samples size.

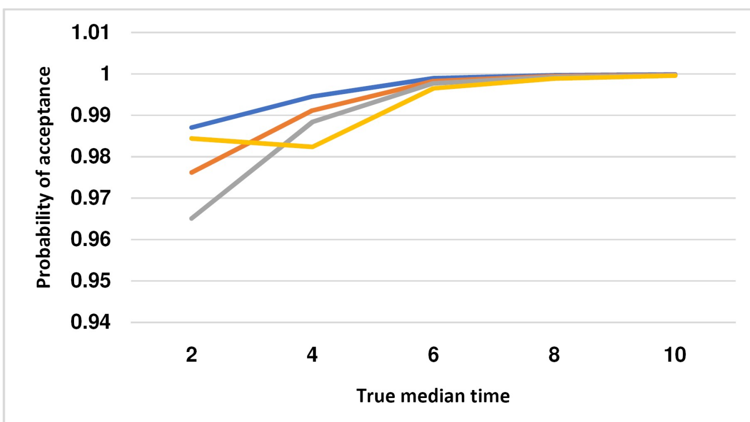

Figure 2. True median time versus the probability of acceptance.

5. Concluding Remarks

The single sampling plan attribute was proposed and designed using the two-point approach under the assumption that the lifetime of a product follows the (P-A-L) extended Weibull distribution. The two-point approach is to determine the plan parameters such as the samples size and the acceptance number. This paper only deals with the (P-A-L) extended Weibull distribution. A further study is required to propose more distributions for special cases of the (P-A-L) extended Weibull distribution and compare the results obtained.

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